A Property Rights Based Consolidation Approach

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Abstract:
We suggest a full consolidation approach that takes into account the property rights structure within the subsidiaries, in particular, the majority requirements on restructurings. Our approach employs a property rights index based on cooperative game theory.
A property rights based consolidation approach✩

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Abstract

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Keywords: consolidation, majority requirements, property rights, cooperative game theory, Shapley value

2010 MSC: 91A12,91B99 JEL: C71, G32, G34, M41, M44

1. Introduction

The Financial Accounting Standards Board (FASB) and the International Accounting Standards Board (IASB) take efforts to harmonize their accounting systems. For example, both standard boards are involved in a current project on the accounting of leases. The proposals of this project aim at property rights based lease accounting (right of use approach) (IASB, 2013). With respect to the development of consistent accounting principles, a property rights based view on other aspects of accounting, consolidation, for example, could be advantageous.

If a company controls one or more other companies, the parent company must present consolidated financial statements. The usual indicator for a controlling financial interest is ownership of more than 50 per cent of the voting rights in the shareholders’ meeting of the subsidiary (Accounting Standard Codification (ASC) 810-10-15-8/International Financial

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Reporting Standard (IFRS) 10.B35). As a consequence, “the assets and liabilities, equity, income, expense, and cash flows of the parent company and its subsidiaries are presented as those of a single economic entity” (International Accounting Standard (IAS) 27.4).

Yet, the parent’s share of the potential outcome of the subsidiary indicated by the assets reported according to the aforementioned group accounting approach may not reflect the actual allocation of property rights in the subsidiary. In particular, these property rights are crucially influenced by the majority requirements imposed on decisions that potentially shift gains from the subsidiary to the parent. Moreover, these requirements vary due to country specific corporate law. So far, little attention has been paid to this fact.

Especially restructurings may enable the majority shareholder of the subsidiary to shift future profits to the parent company. Hence, minority interests may suffer heavy losses due to restructurings. In order to avoid these or similar disadvantages for minorities, the German legislator imposed qualified majority requirements for decisions on restructurings. Property rights of majority shareholders in public corporations in the United States are not restricted by such qualified majority requirements. Therefore, the same majority capital share gives rise to different voting power in American and in German subsidiaries. So, one could argue that this difference should be reflected in the consolidated financial statements of the parent. This way, a potential investor gains a more accurate picture of the risks related to blocking minorities.

Due to the United States Generally Accepted Accounting Principles (US-GAAP) and IFRS, non-controlling interest (NCI) is completely accounted within the equity section (ASC 810-10-45-16/IFRS 10.22). This accounting approach is justified by the entity theory (Baluch et al., 2010, p. 4). In contrast, the parent company theory suggests to account the subsidiary’s assets and liabilities proportionately (Baluch et al., 2010, pp. 2). However, the splitting of assets and liabilities is not amenable to a sound economic interpretation (Schmidt, 2003, p. 145). Moreover, the property rights actually allocated to the parent measured, for example, by its voting power in the shareholders’ meeting may not coincide with its ownership in the subsidiary (see Casajus et al., 2009).

Reporting of NCI either within or outside the equity section might matter in terms of value relevance. Previous studies analyzed the different recognition of essentially the same accounting information in the same financial statement (Cahan et al., 2000; So and Smith, 2009). Particularly, in their investigation of NCI reporting, Lopes et al. (2012) concluded that investors are not sensitive to changes in the reporting location but to the presence of NCI at all. Nevertheless, the nature of NCI as debt or equity is crucially determined by the possibility of blocking minorities due to corporate law, which might differ among countries.

In this paper, we suggest an accounting approach that avoids the shortcomings of the above-mentioned accounting theories. We adhere to the full consolidation method implied by the entity theory. Yet, in general, we do not account the whole NCI within the equity section, but split it between the equity section and the debt section. This split rests on a measure of the actual property rights of the parent based on cooperative game theory.

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1 In the following, we adhere to the simplifying assumption that voting rights coincide with the capital share.
Note that the application of cooperative game theory, in particular, of the Shapley value (Shapley, 1953) already proved to be useful in accounting. While Newman (1981) use this value in order to evaluate the distribution of power in the Accounting Principles Board (APB) and in the FASB, Selto and Grove (1983) employ it to analyze the voting power of the FASB with respect to certain statements of the Financial Accounting Standards.

The paper is organized as follows. In the second section, we provide economic and legal foundations. In the third section, we first develop a property rights index based on cooperative game theory. Second, we propose a modified full consolidation approach that employs this property rights index. Some remarks on limitations of our approach and possible remedies conclude the paper. An appendix contains the calculations concerning the property rights index.

2. Economic and legal foundations

2.1. Basic consolidation approaches

The purpose of consolidated financial statements is to compensate the lack of information inherent in the separate financial statements of a parent and its subsidiaries. For example, separate financial statements can misrepresent the “real” gains within a group due to inappropriate pricing of intragroup transactions. Therefore, all such intragroup transactions must be eliminated as part of the consolidation (ASC 810-10-45-1/IFRS 10.B86(c)). This way, the consolidated financial statements reduce information asymmetries between the shareholders and the management of the parent company and thereby also reduce the risk of a misallocation of capital.

The current method of consolidation according to both US-GAAP and IFRS is the full reporting of all assets and liabilities at fair value in the consolidated financial statements (ASC 805-20-30-1/IFRS 3.18). According to US-GAAP and IFRS, the aggregated NCI in the subsidiary are reported in a separate line in the equity section (ASC 810-10-45-16/IFRS 10.22).

This consolidation method is based on the entity theory, which assumes homogeneous interests between the majority shareholder and the minority shareholders (Baluch et al., 2010, p. 4). Accordingly, the consolidated financial statements are intended to provide information to both the parent company’s shareholders and the subsidiaries’ shareholders as well as to any other stakeholders (Beams et al., 2012, pp. 393).

In contrast, the parent company theory recognizes inhomogenous interests of the majority shareholder and of the minority shareholders. It considers the consolidated financial statements as an extension of the parent company’s statements. Therefore, the addressees of the consolidated financial statements are only the shareholders and stakeholders of the parent company (Beams et al., 2012, pp. 393). The basic idea of accounting due to the parent company theory is to report the share of the parent in the subsidiaries in the parent’s consolidated financial statements. Aspects of this approach can be found in the accounting of shares in joint ventures according to IAS 31.30 (old version). In the course of convergence attempts between the FASB and the IASB, however, proportionate consolidation was
abolished for joint ventures by adopting IFRS 11. Yet, according to ASC 810-10-45-14, proportionate consolidation may apply under certain circumstances.

2.2. The assessment of control and the effect of protective rights

A company shall present a consolidated financial statement when it controls one or more entities. In particular, the investor is exposed to or has the rights to variable returns from its involvement and it is able to affect those returns through its power over the investee (IFRS 10.6). Assessing the requirement of consolidation according to IFRS 10, the investor must analyze the relevant activities of the investee that significantly affect its variable returns (IFRS 10.B11). An essential point is that the investor has existing rights that give it a current ability to direct the relevant activities (IFRS 10.B9), i.e., power can only be based on substantive rights.

According to US-GAAP there are two consolidation models, one for Variable Interest Entities (VIE) and one for voting interest entities. The Variable Interest Model must be used primarily (ASC 810-10-15-3). This approach is similar to the IFRS 10-consolidation-concept. The investor must have the power to direct the VIE’s economic performance significantly. Furthermore, the investor has the obligation or the rights to absorb losses and benefits from the investee (Shamrock, 2012, p. 134-135). The traditional voting interest model is only used, if the investee is no VIE (ASC 810-10-15-8).

In case of copious minority rights, the controlling influence of the majority shareholder can be restricted significantly. The crucial point is that these rights must be substantive and not protective only (IFRS 10.B25). If the rights relate to essential measures out of the ordinary business, for example, restructurings, they are protective and do not oppose controlling influence (IFRS 10.B27/ASC 810-20-25). For example, the ability to block restructurings according to the German Reorganization Act (§ 16 UmwG) does not restrict the controlling influence of the majority shareholder, because these rights do not affect relevant activities in the ordinary course of business. However, temporarily blocked mergers can have a significant impact on future synergy effects and also on variable returns.

A loss of synergy effects due to blocking minorities, for example, might necessitate an economic approach towards control. The Economic Analysis of Law investigates legal standards regarding their economic consequences, in particular, their costs and benefits resulting from future actions (Zeff, 1978, p. 56). Both the property rights approach and the theory of transaction costs (Coase, 1988, p. 35; Posner, 2007, p. 31) are current methods in the context of the economic analysis of law. The property rights approach investigates the controlling rights of goods and resources (Furubotn and Pejovich, 1972, p. 1139). Therefore, the property rights approach might be an appropriate criterion in order to evaluate the controlling influence and also might be useful to predict the impact of current actions on future cash flows. To put it in another way, the analysis of the property rights structure may facilitate the distinction between substantive rights and protective rights. This economic approach resembles the concept of the planned lease accounting (ED 2013/6), because the classification of leasing contracts also depends on the transfer of property rights (right of use approach).
2.3. Property rights, majority rules, and consolidation

The United States Supreme Court describes property as “a collection of individual rights which, in certain combinations, constitute property” (see United States Supreme Court, 2002, p. 278). In this collection, the several rights are (1) the right to use the thing (ius usus), (2) the right to earn income from the thing (ius usus fructus), (3) the right to change form and substance of the thing (ius abusus), and (4) the right to transfer the thing to others (ius successionis) (Furubotn and Pejovich, 1972, p. 1139-1140). Different rights in the same thing can be owned by different parties (Demsetz, 1967, p. 354). Corporate property rights can be divided into the right to use and earn (operating and financial measures) and the right to change and sell (structuring measures).

The central question of the theory of property rights is how the allocation of these “rights in a thing” affects individual behavior and the resulting economic outcome. The better property rights are specified and the more they are concentrated in one hand, the higher are the incentives to an efficient use of the property (Furubotn and Pejovich, 1972, p. 1141; Posner, 2007, p. 32).

In the United States, the majority requirements concerning public corporations are regulated individually by the states. While the general default majority requirement is the simple majority, qualified majorities may be stipulated in the articles of incorporation. In particular, a simple majority requirement applies to restructurings, for example, mergers or resolutions on dissolution. However, according to Section 10.06.546 of the Alaska Corporations Code, two thirds of the outstanding shares must approve mergers, consolidations, or exchanges.

In Germany, a simple majority applies to the appointment of the members of the supervisory board, decisions on the appropriation of the available profits, the formal approval of the acts of members of the supervisory board, and the appointment of the auditors (§ 119 (1) Aktiengesetz (AktG)). Instead, decisions on amendments of the statutes (§ 179 AktG), on capital measures (§ 222 AktG), on integrations of public companies (§ 319 AktG), on mergers (§ 65 Umwandlungsgesetz (German Reorganization Act), or on dissolutions (§ 262 AktG) require a qualified majority of 75 per cent.

The specific majority requirement influences and restricts the property rights of the controlling shareholder. For example, a 75 per cent majority requirement on certain decisions gives rise to blocking minorities in the shareholders’ meeting, which may prevent or delay the implementation of efficiency enhancing measures (see e.g. Casajus et al., 2009, pp. 10). Hence, one could argue that the extent of property rights of the controlling shareholder should be indicated in the consolidated financial statements. A possible way to do this

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2In public companies, shareholders do not own the assets directly. While the company owns all the assets, the shareholders only possess the equity of the company (May, 1986, p. 226).

3See § 7.25 Model Business Corporation Act (revised).

4See § 7.27 Model Business Corporation Act (revised).

5Regarding mergers, resolutions on liquidations, and resolutions to divest, see for example §§§ 903(a)(2), 909(a)(3), 1001 New York Business Corporation Law or §§ 251 Delaware General Corporation Law.

6A qualified majority requirement of 75 per cent also applies to mergers according to 907(1) of the United Kingdom’s Companies Act 2006.
would be to report a part of the minority interest in the debt section, depending on the property rights of the controlling shareholder measured by some index.

3. Consolidation based on property rights

In this section, we introduce an index that measures the extent of property rights of a major/majority shareholder (parent) in a subsidiary that takes into account the majority requirements on restructurings. Based on the property rights index, we suggest a modification of the full consolidation approach.

The property rights index is based on ideas from cooperative game theory, which already have been used to analyze corporations. For example, Leech (1988) considers the shareholders’ voting power in shareholders’ meetings. In particular, he first models these meetings by weighted majority games and then applies the well-known Shapley value (Shapley, 1953) in order to determine shareholders’ individual voting power. This approach widely has been employed to study other committees as the UN Security Council (e.g. Bailey and Daws, 1998) or parliaments (e.g. Rapoport and Golan, 1985; Carreras and Owen, 1988; Felsenthal and Machover, 2001).

We utilize a modified version of weighted majority games that reflects property rights aspects of voting in the shareholders’ meeting of a subsidiary. In order to measure the extent of property rights of a major/majority shareholder (parent), we adhere to the Shapley value.

3.1. Game theoretic foundations

A (TU) game is a pair \((N, v)\) consisting of a non-empty and finite set of players \(N\) and a coalition function \(v \in V(N) := \{f : 2^N \to \mathbb{R} \mid f(\emptyset) = 0\}\), where \(2^N\) denotes the power set of \(N\). Subsets of \(N\) are called coalitions; \(v(S)\) is called the worth of coalition \(S\) indicating what the players in \(S\) can “achieve” if they cooperate. A value on \(N\) is an operator \(\varphi\) that assigns a payoff vector \(\varphi(N, v) \in \mathbb{R}^n\) to any game \((N, v)\), where \(\varphi_i(N, v)\) denotes player \(i\)’s payoff.

The most influential and widely employed value is the Shapley value (Shapley, 1953). It is defined as follows: A rank order on \(N\) is a bijection \(\rho : N \to \{1, \ldots, n\}\), where \(\rho(i)\) is the position of \(i \in N\) in \(\rho\). The set of all rank orders on \(N\) is denoted by \(R(N)\). The set of players up to and including \(i\) in \(\rho\) is denoted by \(P_i(\rho) = \{j \in N : \sigma(j) \leq \sigma(i)\}\). The marginal contribution of \(i\) in \(\rho\) is defined as \(MC_i(\rho, v) := v(P_i(\rho)) - v(P_i(\rho) \setminus \{i\})\). Finally, the Shapley value, \(\text{Sh}_i\), gives player \(i \in N\) the average marginal contribution over all rank orders,

\[
\text{Sh}_i(N, v) = |R(N)|^{-1} \cdot \sum_{\rho \in R(N)} MC_i(\rho, v), \quad v \in V(N), \ i \in N. \tag{1}
\]

Besides the Shapley value there are other values (one-point solution concepts) for TU games, for example, the nucleolus (Schmeidler, 1969) or the Banzhaf-Owen value (Banzhaf, 1965; Owen, 1975). In view of its characterization by Young (1985, Theorem 2), the Shapley value is THE value that measures a player’s own productivity in a game. Thus, the Shapley value is the appropriate value to be applied in the next section.
3.2. A property rights index

We consider a subsidiary with \( n > 1 \) shareholders, modelled by the player set \( N_n := \{1, 2, \ldots, n\} \). Their (relative) shares are given by a weight vector \( w = (w_1, w_2, \ldots, w_n) \in \mathbb{R}^n \) such that \( w_i > 0 \) for all \( i \in N_n \) and \( \sum_{i \in N_n} w_i = 1 \). For \( S \subseteq N_n \), let \( w(S) := \sum_{i \in S} w_i \). There is a qualified majority quota \( q \in [0.5, 1] \) necessary to implement certain types of decisions.

Shareholder 1 is the major shareholder/parent, i.e., \( w_1 > w_i \) for \( i \in \{2, \ldots, n\} \). To simplify exposition, the remaining share of \( 1 - w_1 \) is free floating uniformly, i.e., the other (minor) shareholders have equal shares, i.e., \( w_i = \frac{1 - w_1}{n - 1} \) for \( i \in N_n \setminus \{1\} \). Moreover, there are many minor shareholders, i.e., \( n \) goes to infinity.

Fix \( w_1, w, \) and \( q \) as above. In order to determine the extent of property rights of the major shareholder, we consider a sequence of games \([q, w_1, n] := (N_n, v_n) \), \( n \in \mathbb{N} \), where \( v_n \) is given by\(^7\)

\[
v_n(S) = \begin{cases} 
 1, & w(S) > q \text{ and } 1 \in S, \\
 0, & w(S) < q \text{ and } 1 \notin S \\
 w(S), & w(S) > q \text{ and } 1 \notin S, \\
 w(S), & 1 - q > w(S) \text{ and } 1 \in S, \\
 0, & 1 - q > w(S) \text{ and } 1 \notin S.
\end{cases}
\]

The basic idea behind the coalition functions \( v_n \) is that decisions, to which the quota \( q \) applies, may shift gain generating units from the subsidiary to the major shareholder (parent\(^8\)). A group of shareholders whose share exceeds the quota cannot be prevented by the rest of the shareholders from appropriating the whole (future) gains of the subsidiary (normalized to 1). If this group contains the parent, the latter can compensate for the minor shareholders’ losses that occur by shifting future gains to the parent and thereby convince them to agree to the gain-shifting decision (first line). In case this group does not contain the parent, the group has no interest in shifting gains to the parent and the group ends up with a gain proportional to its share in the subsidiary (second line). Conversely, a group of shareholders whose joint shares fall short of \( 1 - q \) cannot block the gain-shifting decision and may end up with a zero gain, when the parent is not present in this group (fifth line). If the group contains the parent, the shareholders outside this group have no incentive to shift gains to the parent and this group realizes the proportional gain (fourth line). If a group of shareholders cannot shift future gains, but can block such a shift, then this group participates in the future gain proportional to its share (third line).

In order to measure the extent of property rights of the parent, we determine the limit of Shapley payoffs of the parent (player 1) in the games \([q, w_1, n] \) as the number of minor shareholders goes to infinity. The latter indicates that we assume the minor interests to be free floating infinitely fine. First, this assumption renders the parent’s property rights index as great as possible. Hence, our recommendations based on this assumption are rather

\(^7\)Note the games \( v_n \) resemble the ocean games studied by Shapiro and Shapley (1978).

\(^8\)To simplify exposition, we refer to a major shareholder as parent even if its interest in the subsidiary is smaller than 50 per cent.
conservative (see Section 3.3). Second, this simplification leads to a workable formula for the index. From Eq. (1) and Eq. (2), one obtains

\[
PRI_1(q, w_1) := \lim_{n \to \infty} Sh_1[q, w_1, n] = \begin{cases} 
1, & 1 > w_1 > q, \\
w_1 + (1 - q) \cdot \frac{1 - q}{1 - w_1}, & q \geq w_1 > 0,
\end{cases}
\]

for \( w_1 \in (0, 1) \) and \( q \in [0.5, 1] \). The lengthy and technical proof of this result is referred to the appendix.

The number \( PRI_1(q, w_1) \) in Eq. (3) is called the **property rights index** of the parent in a subsidiary, where its relative share is \( w_1 \in (0, 1) \) and the majority requirement for gain-shifting decisions is \( q \in [0.5, 1] \). Let us interpret this index. The first line of Eq. (3) is obvious. Whenever the parent’s share is greater than the quota, the parent can attract all the future gains. The second line in Eq. (3) refers to the situation where the parent needs the support by some of the minor shareholders to shift gains to the parent. The parent’s payoff consists of two parts. First, the parent obtains a gain proportional to its share in the subsidiary. On top of this, the parent is assigned a fraction of the possible shifting gains of \( 1 - q \). The fraction equals the fraction of minor shareholders the parent does not need in order to meet the majority requirement, \((1 - q) / (1 - w_1)\).

Some remarks on the coalition functions \( v_n \) and the property rights index seem to be in order. By construction, the major shareholder (player 1) is treated as parent even if his share is rather small. This makes a major shareholder with a small share extremely powerful. For example, this is reflected by the fact that the property rights index converges to \((1 - q)^2\) as the parents share goes to 0. Hence, the property index should be interpreted with care when the major shareholder’s share indicates that he cannot be considered as a parent.

### 3.3. Consolidation based on the property rights index

According to future lease accounting (IASB, 2013, p. 5), the right of use will be separated from the leased asset and will be reported in the balance sheet of the lessee. Correspondingly, the remaining part of the property rights will be accounted in the balance sheet of the lessor. This right of use approach is a step towards property rights based accounting of leasing contracts.

In the same vein, it might be interesting and appropriate to develop a property rights based accounting approach also for the consolidation of groups. Such an approach may rest on the property rights index suggested in the previous subsection as follows. The total assets of the subsidiary are reported at their fair value in the consolidated financial statements. The non-controlling interest is split and reported in the equity section and in the debt section such that the fraction of the non-controlling interest reported in the equity section plus the share of capital of the parent equals the property rights index, which depends on the majority requirement and the share of the parent in the subsidiary. That is, the fraction of 

\[
\frac{1 - PRI_1(q, w_1)}{1 - w_1} = \begin{cases} 
0, & 1 > w_1 > q, \\
1 - \left( \frac{1 - q}{1 - w_1} \right)^2, & q \geq w_1 > 0,
\end{cases}
\]
of the non-controlling interest is reported in the debt section and the remainder in the equity section. Note, if PRI1(q, w1) = 1, then the total non-controlling interest is accounted within the equity section. In the next subsection, we explain our property rights based full consolidation approach with an example.

In Figure 1, we depict the fraction of the non-controlling interest to be reported in the debt section according to our approach for some relevant majority quotas, 50 percent, 66.7 per cent (two thirds), and 75 per cent, and shares of the parent in the subsidiary ranging from 20 per cent to 100 percent. On the one hand, we disregard shares less than 20 percent, which indicate an insignificant influence, both according to US-GAAP and IFRS (Shamrock, 2012, pp. 156). On the other hand, we consider shares less than 50 per cent because they can be controlling (see IFRS 10.B38).

The diagram shows that, ceteris paribus, a higher majority requirement entails a higher fraction of the NCI to be reported in the debt section, while a higher share of the parent in the subsidiary puts a lower fraction of the NCI in the debt section. This is quite plausible because a higher majority quota or a lower interest decreases the parent’s property rights in the subsidiary.

Alternatively, one could report only the fraction of PRI1(q, w1) of the assets in the consolidated financial statements in order to indicate the property rights structure in the subsidiary. This would be consistent with the right of use approach. Yet, splitting the assets usually is regarded to be hard to interpret.
3.4. An example

Consider a parent company, situated in the United States, that has an interest of 60 per cent both in an American subsidiary and in a German subsidiary. In the United States, the majority requirement for restructurings is 50 per cent; in Germany, it is 75 per cent. This results in a property rights index of \( \text{PRI}_1^\text{US} (0.5, 0.6) = 1 \) in the American subsidiary and of \( \text{PRI}_1^\text{GER} (0.75, 0.6) = 0.7562 \) in the German subsidiary. Accordingly, the total non-controlling interest of the American subsidiary is reported within the equity section. In contrast, the non-controlling interest of the German subsidiary is split and accounted as follows. Assume that both the American subsidiary’s total equity and the German subsidiary’s total equity amounts to US$ 100,000 (including unrealized gains). Hence, the non-controlling interest amounts to US$ 40,000. According to our approach, the amount of

\[
\frac{1 - 0.7562}{1 - 0.6} \times \text{US$ 40,000} = \text{US$ 24,375}
\]

goes to the debt section, while the rest amounting to US$ 15,625 remains the equity section.

The standard full consolidation process and the property right based full consolidation process are illustrated in Table 1 and Table 2), respectively. Table 1 rests on the following eliminations and adjustments: First, the parent’s interests in its subsidiaries have to be eliminated, consolidation adjustments (a) and (b). Second, the non-controlling interests have to be recorded within the equity section, consolidation adjustments (c) and (d). Table 2 results from an additional adjustment due to the property rights based consolidation approach, consolidation adjustment (e).

4. Conclusion

While consolidation standards are quite uniform throughout the world, corporate law significantly differs between countries. Therefore, consolidated financial statements can provide inappropriate information. Corporate law, in particular, majority requirements, crucially affects the distribution of property rights in a public corporation. This is the reason why we propose a full consolidation approach that takes into account the property rights structure in a subsidiary.

The property rights index employed in our approach rests on a strong simplifying assumption on the distribution of the non-controlling interest. Specifically, we assume that the non-controlling interest is free floating. This assumption tends to exaggerate the extent of property rights of the parent, leading to a rather conservative consolidation policy that keeps a great part of the non-controlling interest in the equity section. To remedy this limitation of our approach, one could calculate the property rights index on the basis of the actual distribution of shares among shareholders.
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<td>100</td>
<td></td>
<td>300</td>
</tr>
</tbody>
</table>

Table 1: Full consolidation according to entity theory

<table>
<thead>
<tr>
<th>US$’000</th>
<th>parent (US)</th>
<th>sub. (US)</th>
<th>sub. (GER)</th>
<th>eliminations</th>
<th>group</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Dr.</td>
<td>Cr.</td>
</tr>
<tr>
<td>investment (US)</td>
<td>60</td>
<td></td>
<td></td>
<td>(a) 60</td>
<td>—</td>
</tr>
<tr>
<td>investment (GER)</td>
<td>60</td>
<td></td>
<td></td>
<td>(b) 60</td>
<td>—</td>
</tr>
<tr>
<td>current assets</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
<td>300</td>
</tr>
<tr>
<td>total assets</td>
<td>220</td>
<td>100</td>
<td>100</td>
<td></td>
<td>300</td>
</tr>
<tr>
<td>share capital</td>
<td>220</td>
<td>100</td>
<td>100</td>
<td>(a) 60</td>
<td>220</td>
</tr>
<tr>
<td>NCI (equity)</td>
<td></td>
<td></td>
<td></td>
<td>(b) 60</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(c) 40</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(d) 40</td>
<td></td>
</tr>
<tr>
<td>current liabilities</td>
<td></td>
<td></td>
<td></td>
<td>(e) 24.375</td>
<td>55.625</td>
</tr>
<tr>
<td>NCI (debt)</td>
<td></td>
<td></td>
<td></td>
<td>(e) 24.375</td>
<td>24.375</td>
</tr>
<tr>
<td>total equity &amp; liabilities</td>
<td>220</td>
<td>100</td>
<td>100</td>
<td></td>
<td>300</td>
</tr>
</tbody>
</table>

Table 2: Full consolidation according to property rights
Appendix A. Calculations to Eq. (3)

For \( \rho \in R(\mathbb{N}_n) \), set \( \rho_1 := \rho(1) \). By Eq. (2), we have

\[
MC_1(\rho, v_n) =
\begin{cases}
(i) & 1, \\
(ii) & 1 - (\rho_1 - 1) \cdot \frac{w_1}{1 - w_1}, \\
(iii) & w_1 + (\rho_1 - 1) \cdot \frac{w_1}{1 - w_1}, \\
(iv) & w_1, \\
(v) & 1 - (\rho_1 - 1) \cdot \frac{w_1}{1 - w_1}, \\
(vi) & w_1 + (\rho_1 - 1) \cdot \frac{w_1}{1 - w_1},
\end{cases}
\]

The conditions in (A.1) can be rewritten as follows

\[
\begin{align*}
(i) & \frac{1-q}{1-w_1} \cdot (n-1) + 1 > \rho_1 > \frac{q-w_1}{1-w_1} \cdot (n-1) + 1, \\
(ii) & \frac{q}{1-w_1} \cdot (n-1) + 1 \geq \rho_1 > \frac{1-q}{1-w_1} \cdot (n-1) + 1, \\
(iii) & \frac{q-w_1}{1-w_1} \cdot (n-1) + 1 \geq \rho_1 \geq \frac{1-q}{1-w_1} \cdot (n-1) + 1, \\
(iv) & \frac{1-q}{1-w_1} \cdot (n-1) + 1 \geq \rho_1 \geq \frac{q-w_1}{1-w_1} \cdot (n-1) + 1, \\
(v) & \rho_1 > \frac{q}{1-w_1} \cdot (n-1) + 1, \\
(vi) & (n-1) \cdot \frac{1-q}{1-w_1} + 1 \geq \rho_1.
\end{align*}
\]

If \( w_1 > q \), then \( MC_1(\rho, v_n) = 1 \) for all \( \rho \in R(\mathbb{N}_n) \). By (1), we already obtain the first line of Eq. (3). Let now \( q \geq w_1 \). Since \( q \geq 1 - q \), we have to consider another four cases.

- **Case A**: \( q \geq w_1 \geq 1 - q \), \( 1 - q > q - w_1 \). Non-empty ranges in (A.2): (i), (ii), (iii).
- **Case B**: \( q \geq w_1 \geq 1 - q \), \( q - w_1 \geq 1 - q \). Non-empty ranges in (A.2): (ii), (iii), (iv).
- **Case C**: \( 1 - q > w_1 \), \( 1 - q > q - w_1 \). Non-empty ranges in (A.2): (i), (ii), (iii), (v), (vi).
- **Case D**: \( 1 - q > w_1 \), \( q - w_1 \geq 1 - q \). Non-empty ranges in (A.2): (ii), (iii), (iv), (v), (vi).

We deal with Case A only. The other cases can be handled analogously.

**Case A**: The ranges in (A.2) become

\[
\begin{align*}
(i) & \frac{1-q}{1-w_1} \cdot (n-1) + 1 > \rho_1 > \frac{q-w_1}{1-w_1} \cdot (n-1) + 1, \\
(ii) & \frac{q}{1-w_1} \cdot (n-1) + 1 \geq \rho_1 \geq \frac{1-q}{1-w_1} \cdot (n-1) + 1, \\
(iii) & \frac{q-w_1}{1-w_1} \cdot (n-1) + 1 \geq \rho_1 \geq \frac{1-q}{1-w_1} \cdot (n-1) + 1.
\end{align*}
\]

As \( n \to \infty \), the probabilities of a rank order to fit one of these ranges are (i) \( \frac{1+w_1}{1-w_1} \), (ii) \( \frac{q-w_1}{1-w_1} \), and (iii) \( \frac{q-w_1}{1-w_1} \). Moreover, as \( n \to \infty \), the average marginal contributions \( MC_1(\rho, v_n) \) for rank orders fitting these ranges are (i) \( \frac{q-w_1}{1-w_1} \), (ii) \( \frac{q-w_1}{2} \), and (iii) \( \frac{q-w_1}{2} \). Hence, we have

\[
\lim_{n \to \infty} Sh_1[q, w_1, n] = \frac{1+w_1}{1-w_1} \cdot \frac{1}{2} + \frac{q-w_1}{1-w_1} \cdot \frac{q+w_1}{2} + \frac{q-w_1}{1-w_1} \cdot \frac{q+w_1}{2} = w_1 + (1-q) \cdot \frac{1-q}{1-w_1}.
\]
References


